Single Stage Plant-Warehouse Location Problem (SSPWLP): A State of Art Formulation

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Abstract

Kaufman et. al. (1977) first considered this problem (SSPWLP). Goods flow from plants to warehouses to markets. Here we need to locate plants and warehouses of appropriate capacities so that sum total of location cost of plants and warehouses and distribution cost of goods from plant to warehouses to markets is minimized. They considered normalized decision variables (see Sharma and Muralidhar (2009)). However, they used the formulation style of Geoffrion and Graves (1974). We use normalized variables but use the variable style of Sharma (1991) and Sharma and Berry (2007) that reduces the number of variables. We give several strong linking constraints by drawing from the works of Sharma and NAMDEO (2005) and Sharma and Berry (2007). Below we give the formulation in brief. Details can be seen in the body of the paper. Variables names are self-explanatory and makes understanding the model easier.

Key Words: single stage plant warehouse location problem; location-distribution problem; locating plant and warehouses simultaneously; simple plant location problem and capacitated plant location problem

New Formulation of SSPWLP:

Min sum(i,j), cpw(i,j)*xpw(i,j) + sum(j,k), cwm(j,k)*xwm(j,k) Sum(i), yp(i)*fp(i) + sum(j), yw(j)*fw(j) (0) Sum(i,j), xpw(i,j) = 1 (0a) Sum(j,k), xwm(j,k) = 1 (0b) Sum(j), xwm(j,k) >= d(k) for all k (0c) Sum(j), xpw(i,j) <= capp(i) for all i (1) Sum(j), xpw(i,j) <= capp(i) for all i (1) Sum(j), xpw(i,j) <= capp(j) for all j (3) Sum(i), xpw(i,j) <= capw(j) for all j (3) Sum(i), xpw(i,j) <= capw(j) for all i, j (5) xpw(i,j) <= yw(j)*capp(i) for all i, j (6) xwm(j,k) <= d(k)*yw(j) for all j,k (7)

Sum(j), xpw(i,j) <= yp(i) for all i (8) Sum(i), xpw(i,j) <= yw(j) for all j (9)

Flow balance constraint:

Sum(i), xpw(i,j) = sum(k), xwm(j,k) for all j (10)

Sum(i), capp(i)*yp(i) >= 1 (11)

Sum(j), capw(j)*yw(j) >= 1 (12)

 $xpw(i,j) \ge 0$ for all i,j; $xwm(j,k) \ge 0$ for all j,k (13)

yp(i) = (0,1) for all i and yw(j) = (0,1) for all j (14)

This (the above formulation of SSPWLP) is the best formulation of SSPWLP that is amenable to solution by LP relaxation and attendant branch and bound and/or branch and cut solution procedure. In literature the distribution phase between plant and warehouse (where plant and warehouse are to be located) is solved by Sharma and Agarwal (2014) as MID_CPLP were Lagrangian Relaxation (LR)

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was deployed to get RHS_CPLP and LHS_CPLP (different classes of capacitated plant location problems) that were attempted by wellknown relaxations (LR and Linear Programming) already available in literature (see Priyanka Verma and Sharma (2011)). Computational investigation is underway to determine efficacy of different new linking constraints given in this paper.

1.Introduction

Location-distribution problems have lot of varieties: SPLP (simple plant location problem), CPLP (capacitated plant location problem), single stage capacitated warehouse location problem (SSCWLP), and two stage warehouse location problems (see Sharma and Agarwal (2014) for latest references TSCWLP). In most of these problems plants and warehouses were not located simultaneously. Closest to problem considered in this paper is the problem of 2-stage warehouse location problem where warehouses are located in successive stages (but plant was located already, see Sharma and Namdeo (2005) and Sharma and Pritee Agarwal et. al. (2012, 2013, 2014, 2016)). For latest in plant-warehouse location theory refers to Sharma (2019, 2019, 2020, 2021 and 2022). Here we describe SPLP, CPLP, SSCWLP and TSCWLP in brief.

Problem SPLP is to locate plants of unlimited capacities so that sum total of fixed cost of plants plus distribution cost of plants to markets (so that their demand is met) is minimized. Here plants supply goods directly to markets. Problem CPLP is similar to SPLP but plants have finite capacities. Ordinary decision variables are X(i,k) which denoted the absolute quantity transported from plants to markets. If demand at a market k is D(k) then we have x(i,k) = X(i,k)/sum(k), D(k) and x(i,k) is referred to as normalized distribution variable and it is between 0 and 1. It has several advantages, see Sharma and Muralidhar (2009) for details. In literature SPLP and CPLP were for single commodity and single period only.

In SSCWLP, plants are already located, we need to locate warehouses in between plants and markets. Here goods flow from plants to warehouses to markets. Here if we consider single commodity then 'normalized' distribution variables are useful, but for multi commodity case 'normalized' distribution variables are not useful. Geoffrion and Graves used the distribution variable Y(i,j,k) to denote absolute

quantity of goods transported from plant 'i' to warehouse 'j' to market 'k'. this style did not require flow balance constraints (see Geoffrion and Graves (1974). However Sharma (1991) used transportation variables as XPW(i,j) (quantity transported from plant 'i' to warehouse 'j') and XWM(j,k) (quantity transported from warehouse 'j' to market 'k'. it is easy to see that Sharma (1991) formulation has less decision variables compared to number of decision variables in Geoffrion and Graves (1974) but require additional flow balance constraints at each of the warehouses. Sharma and Berry (2007) who gave several 'strong' constraints (both supply and demand side) and showed that Sharma (1991) type formulation was significantly superior to formulation style of Geoffrion and Graves (1974). Sharma and Berry (2007) considered only a single product and single period problem; whereas Sharma (1991) and Geoffrion and Grave (1974) developed models that were capable of considering multi-products and multi- period cases. Geoffrion and Graves (1974) assumed that whatever came in at a warehouse was moved out and there was no provision of keeping inventory at warehouses in their model. Sharma (1991) considered a model where inventory was allowed to kept at warehouses (but not at markets) and shortages were not allowed. Sharma (2019, 2019, 2020, 2020, 2021 and 2022) have developed models that allow inventory at markets and warehouses and allow shortages at markets.

Sharma and Agarwal (2014) and Sharma and Verma (2011) relaxed flow balance constraints in SSCWLP at warehouses to initiate Lagrangian Relaxation procedure and solved RHS and LHS CPLPs that resulted.

In this paper we apply all the advances listed above to the Single Stage Plant Warehouse Location Problem (SSPWLP) and give its most modern formulation below.

2. Problem Formulation of SSPWLP

It has been well established in literature (see Sharma and Berry (2007) that formulation style of Geoffrion and Graves (1974) is not efficient; and hence that is not given in this paper. We use the style of Sharma (1991) that was demonstrated to be efficient by Sharma and Berry (2007). Since it is the case of single commodity, we use the normalized decision variables (see Sharma and Muralidhar (2009) and Sharma and Berry (2007) and Kauffman et. al (1977).

Index:

'i' for plant, 'j' for warehouse and 'k' for market.

Constants of the Problem

capp(i) is capacity of plant 'i' as a fraction of total market demand; capw(j) is capacity of warehouse 'j' as a fraction of total market demand; d(k) demand at market 'k' as a fraction of total market demand (d(k) = D(k)/sum(k), D(k)); cpw(i,j) is the cost of transporting sum of all market demand from plant 'i' to warehouse 'j' and cwm(j,k) is the cost of transporting sum of all market demand from warehouse 'j' to market 'm'.

Variables of the Problem

xpw(i,j) is the quantity transported from plant 'i' to warehouse 'j' as a fraction of total market demand (xpw(i,j) = XPW(i,j)/sum(k), D(k)); xwm(j,k) is the quantity transported from warehouse 'j' to market 'k' as a fraction of total market demand (xwm(j,m) = XWM(j,m)/sum(k), D(k)); yp(i) is location variable for plant 'i', it is equal to 1 if plant is located at 'i' and 0 otherwise; and yw(j) is location variable for warehouse 'j', it is equal to 1 if warehouse is located at 'j' and 0 otherwise.

New Formulation of SSPWLP:

Min sum(i,j), cpw(i,j)*xpw(i,j) + sum(j,k), cwm(j,k)*xwm(j,k) Sum(i), yp(i)*fp(i) + sum(j), yw(j)*fw(j) (0) Sum(i,j), xpw(i,j) = 1 (0a) Sum(j,k), xwm(j,k) = 1 (0b) Sum(j), xwm(j,k) >= d(k) for all k (0c) Sum(j), xpw(i,j) <= capp(i) for all i (1) Sum(j), xpw(i,j) <= capp(i) for all i (1) Sum(i), xpw(i,j) <= capp(i)*yp(i) for all i (2) Sum(i), xpw(i,j) <= capw(j) for all j (3) Sum(i), xpw(i,j) <= capw(j)*yw(j) for all j (4) xpw(i,j) <= yp(i)*capw(j) for all i, j (5) xpw(i,j) <= yw(j)*capp(i) for all i, j (6) xwm(j,k) <= d(k)*yw(j) for all j,k (7) Sum(j), xpw(i,j) <= yp(i) for all i (8) Sum(i), xpw(i,j) <= yw(j) for all i (9)

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Flow balance constraint:

Sum(i), xpw(i,j) = sum(k), xwm(j,k) for all j (10)

Sum(i), capp(i)*yp(i) >= 1 (11)

Sum(j), capw(j)*yw(j) >= 1 (12)

 $xpw(i,j) \ge 0$ for all i,j; $xwm(j,k) \ge 0$ for all j,k (13)

yp(i) = (0,1) for all i and yw(j) = (0,1) for all j (14)

Equation (0) is sum of cost of location and distribution. Equations (0a), (0b) and (0c) ensure that demand is met at all markets. Eq. (1) ensures flow out of a plant to be within its capacity, Eq. (2) is a strong linking constraint, Eq. (3) ensures that inflow at a warehouse is within its capacity, and Eq. (4) is again a strong linking constraint. Equations (5) and (6) are new linking constraints to SSPWLP but are borrowed from Sharma and Namdeo (2005). Again, equations (7) are strong linking constraints borrowed from literature (Sharma and Berry (2007)). Eqs. (8) and (9) are weak linking constraints, (10) is flow balance constraint at each of the warehouses. Equations (11) and (12) ensure that we have an additional constraint to ensure a feasible solution to problem SSPWLP. These are missed in Kaufman et. al. (1977) and Sharma and Berry (2007); but Priyank Dubey (2020) established its efficacy. Equations (13) force non-negativity restriction on transportation variables and equations (14) ensure that location variables are (0,1).

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Few comments are in order here. Sharma and Namdeo (2005) did not give constraints (0a), (0b), (11) and (12) in their formulation for 2-stage warehouse location problem; and Sharma and Berry (2007) forgot to include constraint (12) in their formulation of SSCWLP (single stage capacitated warehouse location problem) and Sharma, Jha and Priyank (2023) (a paper submitted to IEOM 2023 Houston conference) showed that efficacy of (12) in problem SSCWLP was highly significant. We put all these classes of constraints for SSPWLP to give its state-of-art formulation. It is also important to note here that despite stronger LP relaxation bound given by strong constraints ((2) and (7)) compared to LP relaxation bounds given by weak constraints (8) and (9), SSUWLP (single stage un-capacitated warehouse location problem) ran faster with weak constraints than the strong constraints (see Sharma and Verma (2012)). This encouraged Sharma and Verma (2012) to develop a hybrid formulation (here weak constraints were augmented by few promising strong constraints) whose performance was better than that of weak and strong formulation of SSCWLP. Thus, determination of efficacy of each of the linking constraints is important an experimental investigation is underway to determine this.

3. Discussion

We give the usefulness of the formulation given above in the form of a table.

SN	Advantage
1	New linking constraints (5) and (6) are given that is expected to boost the performance of optimizing algorithms.
2	New feasibility constraints (11) and (12) are expected to boost the performance of optimizing algorithms.
3	We ensure that all strong linking constraints given in Sharma and Berry (2007) (4 and 7) areincluded in this formulation.
4	New linking constraint (2) is added to formulation that is expected to boost the performance of optimizing algorithms.

Table 1: Advantages of New Formulation of SSPWLP given in this paper.

We define following models:

P1: without (5), (6), (11) and (12).

P2: without (11) and (12)

And model P3 with all constraints (0a), (0b), (0c) and (1) to (14).

An experimental investigation carried out to establish the efficacy of new constraints added to SSPWLP problem in this paper is given below. We solved problems for 50 plants and 50 warehouses. In problem set 1 we solved

25 problems with overcapacity between 125–150%. In problem set 2 we solved 25 problems with overcapacity of about 200%. These problems were solved on Intel(R) Core (TM) i5-7200U CPU @ 2.50GHz processor.

4. Computational Results

PROBLEM SET 1 (overcapacity of plant and warehouse between 125%-150%)

Salient Result for P1-P2:

Criterion	μ _{p1}	μ _{p2}	t –value
Iterations	9197.12	6161.64	1.553
No. of Nodes	926.76	625.16	1.528
Root Relaxation Solution Time	0.0328	0.0536	7.076
Objective Fn.	40220.2836	40512.5464	0.337
Execution time	0.0236	0.02872	1.059

Salient Result for P2-P3:

Criterion	μ _{p2}	μ _{p3}	t –value
Iterations	6161.64	1792.80	4.704
No. of Nodes	625.16	74.68	5.112
Root Relaxation Solution Time	0.0536	0.0388	5.220
Objective Fn.	40512.5464	40540.9728	0.039
Execution time	0.02872	0.0312	0.18

Salient Result for P1-P3:

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Criterion	μ _{p1}	μ _{p3}	t –value
Iterations	9197.12	1792.80	3.695
No. of Nodes	926.76	74.68	4.287
Root Relaxation Solution Time	0.0328	0.0388	1.964
Objective Fn.	40220.2836	40540.9728	0.384
Execution time	0.0236	0.0312	2.69*
Sig at 0.0064			

From above table we can say that between P1 and P2 there is no significant difference in terms of number of iterations and no of nodes processed; so, we can say that P1 is as good as P2.

Salient Result for P2-P3:

Criterion	μ _{p2}	μ _{p3}	t –value
Iterations	7456.60	1274.36	3.352
No. of Nodes	568.80	11.72	4.034
Root Relaxation Solution Time	0.0556	0.0552	0.110
Objective Fn.	22921.6592	22422.4352	0.848
Execution time	0.02816	0.03952	1.551*

Salient Result for P1-P3:

Criterion	μ _{p1}	μ _{p3}	t –value
Iterations	7464.68	1274.36	4.577
No. of Nodes	631.76	11.72	5.662
Root Relaxation Solution Time	0.0292	0.0552	8.362
Objective Fn.	23365.4344	22422.4352	1.491
Execution time	0.02876	0.03952	1.551*
Sig at 0.067			

From above table we can say that there is no significant difference in objective function values of models P1, P2 and P3 (except that P3 gives sig better objective functions than P1); but P1 takes significantly less execution time compared to P3. Therefore, we can solve problem first by using model P1 and then use this advanced start to improve further by using model P3 (GAMS offers such a capability).

As more constraints are added (5, 6, 11 and 12) progressively in model P2 and P3 we get better objective function values and at the expense of higher execution time.

5. Conclusion

Priyank Dubey (2020) has established the importance of feasibility constraints such as (11) and (12). Sharma and Verma () have established the importance of Hybrid formulations (weak constraints + most promising strong constraints). Thus, we may construct a model P4 (min (0), s.t. (0a) to (4); (8) and (9); (10) to (14) and most promising constraints associated with strong linking constraints such as (5) to (7). It is expected that this model may give best results for large sized problem instances of SSPWLP. This is a topic of future research.

Declarations

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Biographies

Prof. RRK Sharma: He is B.E. (mechanical engineering) from VNIT Nagpur India, and PhD in management from I.I.M., Ahmedabad, INDIA. He

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