Algorithm to Generate Target for Anti-Lock Braking System using Wheel Power

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Abstract

This paper discusses creating a suitable reference for the Anti-Lock Braking (ABS) control system that maximizes power recovery or dissipation of the brake system. Current anti-lock brake systems implement a finite-state method algorithm to respond to the wheel longitudinal slip by keeping the slip at the given target to maximize braking force. Variables in the logic can include wheel-speed measured from sensors, brake target slip which is estimated by the Electronic Control Unit (ECU), reference speed, which is also estimated, and vehicle acceleration/deceleration. Angular acceleration and calculated actual wheel slip are used as indicators of the wheel’s state of motion. To find the maximum braking force, a typical ABS over-brakes such that the tire longitudinal force exceeds its maximum value and therefore, the wheel starts locking up. The ABS then adjusts by under-braking, and repeats this cycle to keep the tire longitudinal force around its maximum. This paper uses the power absorbed by the wheel to find the maximum power dissipated. Using this strategy, the ABS utilizes continuous control and can better approximate the maximum brake force without the aforementioned cyclic method that has been used in the conventional ABS. In addition to maximizing the brake force as a result of this approach, driver comfort also increases due to the continuous control vs. on/off (cyclic) control during an ABS event.

The first part of the paper consist of modeling the system. Then, using the dynamics equations of the model, we will lay the groundwork of the use of dissipated power in the context of ABS continuous control. The paper then continues with the simulation results (done in MATLAB and Simulink) and follows with the discussion of the results.

Vehicle and Brake model

Figure 1 shows a single wheel tire and all the forces, moments and velocities while braking. This simple model is chosen for the purpose of initial results on the ABS algorithm proposed in this paper. [4].

![Figure 1: Schematic of forces and torques on the wheel](image)

Equations 1 and 2 describe the dynamic equations for this single wheel tire model.

$$J_w \ddot{\omega}_w = F_b R_w - \tau_b$$  \hspace{1cm} (1)

$$M \ddot{u} = -F_b$$  \hspace{1cm} (2)

Where $M$ is the vehicle’s mass, $m$ is wheel’s mass, $F_b$ is the braking force, $N$ is normal force from the ground, $u$ is vehicle’s forward velocity, $\omega$ is angular velocity of the wheel, $J_w$ is wheel’s moment of inertia, $R_w$ is wheel’s radius, $\tau_b$ is the brake torque. For the $F_b$, longitudinal tire force, we can write

$$F_b = \mu \cdot N$$  \hspace{1cm} (3)

Where $\mu$ represents the longitudinal friction coefficient between the tire and the ground. There are different models for the tire forces such as the Finite-Element Method (FEM) and the Pacejka model.
as Pacejka magic tire formula [5], Dugoff [6], LuGre [7], and Burckhardt [8]. In this paper, we choose a simplified Burckhart model to represent the longitudinal tire model, for the sake of its simplicity and being able to capture longitudinal tire saturation well. The longitudinal friction coefficient is defined as

$$\mu = c_1 \cdot (1 - e^{-c_2 \lambda}) - c_3 \lambda$$

where $c_1$, $c_2$, and $c_3$ are tire constants that depend on the road surface type (dry asphalt, wet asphalt, snow, ice) and $\lambda$ is wheel slip and during the brake it is defined as

$$\lambda = \frac{u - R \omega}{u}$$

This relationship (Longitudinal friction Coefficient vs. Slip) is shown for a few of the road surfaces in Figure 2.

A brake actuator was also implemented similar to the one discussed in [9]. This actuator resembles the dynamics of an Electro-Hydraulic brake. The input to the system is the pressure input on the brake fluid and the output is the caliper position which then translates to the brake. The input to the system is the pressure input on the brake fluid [9]. This actuator resembles the dynamics of an Electro-Hydraulic brake actuator was also implemented similar to the one discussed in

Figure 2: Longitudinal Friction Coefficient vs. Slip for different road surfaces using Burckhart tire model

Power Method

The overall control architecture of new continuous ABS algorithm method is shown in Figure 4. The main idea in the this architecture lies under the "Power Method" block. It provides the brake torque reference to the control loop. Additionally, a low-level controller was designed for the brake system using Youla-Kucera robust control technique [4][10].

An overall schematic of the power method reference torque generator is given in Figure 3. The inputs to the Power Method reference generator are wheel angular velocity, the brake torque, and brake torque rate of change. The wheel angular velocity can be obtained through the wheel rate sensor. However, the brake torque is not readily available with the current sensors in a commercial vehicle. Therefore, estimation techniques can be utilized to obtain the estimated brake torque. The output of the reference generator is torque reference ($\tau_{ref}$). We will discuss about $\frac{d\tau_{ref}}{dt}$ later in this section and also in the Mathematical Background section. Writing a simple power balance for the wheel yields to Equation 7.

$$\text{Power} = \dot{\text{P}}_\omega \omega + \tau_\omega \omega + F_b (u - R_{eff} \omega) = F_b u$$  \hspace{1cm} (7)

Figure 3: Power Method Reference Torque Generator for ABS

Where $\dot{\text{P}}_\omega$, $\omega$, $\tau_\omega$, $F_b$, and $R_{eff}$ are the derivative of angular momentum ($\dot{J}_p = J_p \omega$), wheel angular velocity, brake torque, brake force, vehicle speed and effective wheel radius, respectively. A steady state case is considered where there is no power exchange due to the tire inertia. This is a good approximation of operating in the linear region of the friction-slip curve where saturation momentum has less of an effect. Therefore, the power balance would result in Equation 8.

$$\text{Power dissipated} = \tau_\omega \omega = F_b R_{eff} \omega$$  \hspace{1cm} (8)

Equation 8 shows that braking force of the vehicle is associated with the power dissipated by the brakes on wheel. If the power dissipated by the brakes is maximized, the braking force is also maximized. This happens near the peak of the friction-slip curve. If the slip becomes greater than the maximum slip, this would result in the wheel lock up (and therefore zero angular velocity of the wheel) and the power would immediately go to zero. To search for maximizing the dissipation power, the algorithm shown in Figure 3 is used. It compares the previous power dissipation with the current power dissipation and if that power is increasing, then the dissipated power still hasn’t reached its maximum. Therefore, the wheel torque brake can increase. Once the previous power dissipation is more than the current power dissipation, the maximum power dissipation has been reached and therefore, the previous brake torque is the maximum torque brake that can be exerted by the brake. Brake torque increase/decrease in each step is called $\frac{d\tau}{dt}$ (also referred as torque rate in this paper). This variable plays a very important role in this heuristic search as we will discuss in the following sections. We will discuss different ways to find a solution for an optimal $\frac{d\tau}{dt}$ as it is a very important variable in the power method algorithm. Loyola et. al has suggested a 2-D lookup table based on acceleration and velocity of the vehicle for this variable. [4] In this paper, for more effectiveness and robustness of the method, we use an adaptive torque rate ($\frac{d\tau_{ref}}{dt}$) during the ABS event.

Figure 4: Overall control architecture of the new ABS method
Rate of Torque Calculation Using Coefficient of Friction

Linearizing the relationship between longitudinal force and longitudinal slip velocity requires,

\[ F_b = \frac{\partial F_b}{\partial u_s} u_s + F_{b0} \]  \hspace{1cm} (9)

where \( u_s \) is the longitudinal slip velocity and is equal to

\[ u_s = u - R_w \omega_w \]  \hspace{1cm} (10)

And in the linear portion of the curve \( F_{b0} = 0 \). If we differentiate the wheel spin dynamics of Equation 1 with respect to time and substitute Equation 9; then we can write,

\[ J_w \frac{d \omega_w}{dt} = \frac{d}{dt} \left( \frac{\partial F_b}{\partial u_s} u_s R_w \right) - \frac{d \tau_b}{dt} \]  \hspace{1cm} (11)

Hence, we can simplify Equation 11 and approximate \( \frac{d \tau_b}{dt} \) by the following equation,

\[ \frac{d \tau_b}{dt} = NR_w \frac{\partial u_s}{\partial u_s} \frac{du_s}{dt} \]  \hspace{1cm} (12)

where in approximating Equation 11, we assumed that the angular jerk is relatively small compared to the other terms of Equation 11, the tire radius and \( \frac{\partial F_b}{\partial u_s} \) are approximately constant and do not change with time, and \( F_b = \mu N \), where \( N \) is the tire normal force and it is assumed that the normal force does not change with time (it has a slower dynamics than the wheel dynamics). Now, if we substitute Equation 5 into Equation 12 and replace \( \omega \) by \( \omega \) as \( \omega = \frac{\dot{\omega}}{\dot{\omega} \omega} \), provided an initial slip is tested on different values \( \mu \) and \( \omega \). And if the value of rate of torque is set low, it will result in the peak of the power happen at a later time. It takes longer for the algorithm to find the steady-state solution since at the different speeds on a dry asphalt with the initial slip of 0.7. It takes the maximum friction coefficient without even getting near the locking of the wheel at any point (Figures 8 and 7). Figure 9 shows the coefficient of friction \( \mu \) performs well in different road surfaces, initial velocities, and initial slips. Figures 6 and 5 show the coefficient of friction \( \mu \) which was chosen by many researchers such as [11],[12],[13],[14], and [15]. This force estimation along with the derivative would inherently add delays to this signal. Therefore, we have used a 10 milliseconds delay for this signal.

In the next section, we present a few simulation results due to implementation of the adaptive \( \frac{d \mu}{dt} \) power method using a rate of torque being calculated by the derivative of longitudinal tire force and its comparison with a constant rate of torque.

Rate of Torque Calculation Using Longitudinal Tire Force

Assuming that \( \omega_w \) is zero during the steady-state parts of the brake, from Equation 1, we write

\[ \tau_b = R_w \cdot F_b \]  \hspace{1cm} (19)

Then \( \frac{d \tau_b}{dt} \) would become

\[ \frac{d \tau_b}{dt} = R_w \frac{d F_b}{dt} \]  \hspace{1cm} (20)

Which means that \( \frac{d \mu}{dt} \) can be calculated from the derivative of the tire longitudinal force. This would require an estimation of tire’s longitudinal force which has been research by many researchers such as [11],[12],[13],[14], and [15]. This force estimation along with the derivative would inherently add delays to this signal. Therefore, we have used a 10 milliseconds delay for this signal.

Results and Discussion

In this section, we discuss the results of the power method ABS algorithm which is proposed in the previous sections. The initial conditions for the simulations are different initial velocity of the vehicle, different road surfaces, and an initial slip which accounts for the time that ABS starts its process which is usually after moments of hard braking by the user. The initial slip is tested on different values (0.3 and 0.7) to ensure that the ABS power method algorithm works for initial slips near wheel lock (\( \lambda = 1 \)) and smaller values which are still far from wheel lock.

Figures 5, 6, 7, 8 and 9 show the results of the adaptive \( \frac{d \mu}{dt} \) using the derivative of longitudinal force. As shown in the Figures, this method performs well in different road surfaces, initial velocities, and initial slips. Figures 6 and 5 show the coefficient of friction \( \mu \) in different conditions. The maximum coefficient of friction is also plotted for the comparison. As shown in these figures, the vehicle is really close to the maximum friction coefficient without even getting near the locking of the wheel at any point (Figures 8 and 7). Figure 9 shows the reference brake torque generated using the power method at different speeds on a dry asphalt with the initial slip of 0.7. It takes longer for the algorithm to find the steady-state solution since at the higher velocities, the power dissipated would be larger; and therefore, the peak of the power happen at a later time.

As seen in Figures 11 and 12, a constant \( \frac{d \mu}{dt} \) can become sensitive over different conditions and will lock-up the wheel if this constant value is too high under certain road surfaces, initial velocities, and initial slips. And if the value of rate of torque is set low, it will result
in a sub-optimal friction which would result in a longer stopping distance. Two different values of rate of torque has been tested and compared with the adaptive rate of torque which uses the derivative of longitudinal force.

As show in the Figures, this constant value is dependent on the road surface, initial velocity and initial slip that the ABS algorithm initiated which would require a look-up table and a lot of calibration and tuning during the implementation. In all the cases, the adaptive rate of torque shows a better performance (higher friction coefficient, \( \mu \)) and does not lock the wheel. Figure 3 compares the dissipated power for to cases of constant rate of power and the adaptive rate of power for an initial velocity of 40 mph on a wet asphalt with the initial slip of 0.7. As we can see from Figure 11, when \( \frac{d\tau}{dt} = 200 \), the wheel locks up. This can be seen in Figure 3 as well since the dissipated power doesn’t reach its peak and goes to zero before the other two cases. When \( \frac{d\tau}{dt} = 200 \), although it doesn’t lock the wheel, and it looks like an optimal solution from a power point of view, the steady-state value of coefficient of friction doesn’t stay close to the optimal solution and decreases. The only explanation for this would be that because of the nature of constant torque rate, once the algorithm reached one step before the peak of power (which would translate to the optimal brake torque), it then added one more constant step to the near optimal brake torque and it caused it to go over the peak of power; therefore, the coefficient of friction immediately dropped. This just proves what we discussed earlier regarding the constant rate of torque being hard to tune.
Conclusion

We presented an ABS target generator using the dissipating power of the wheel for a continuous ABS control scheme. Different methods were used to find the solution for a rate of torque that gives the best results. An adaptive rate of torque using the derivative of longitudinal force was used and showed promising results.
References


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